REPORT DOCU**ENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)* 01-09-2005	**2. REPORT TYPE** Research	**3. DATES COVERED** *(From - To)* April 2003 - Sept 2005

4. TITLE AND SUBTITLE
Free space measurements of negative refraction with varying angles of incidence

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
J. Derov, B. Turchinetz*, E. Crisman**, A. Drehman, S. Best and R. Wing

5d. PROJECT NUMBER
2305

5e. TASK NUMBER
HA

5f. WORK UNIT NUMBER
02

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Source code 437890

AFRL/SNHA
80 Scott Dr.
Hanscom AFB 01731-2909

* contractor for CNS Inc.
702 King Farm Blvd, Rockville MD
**contractor for Univ. of RI
Dept Chem E, Crawford Hall,
Kingston RI 02881

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
-

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
AFRL-SN-HS-JA-04-0316

12. DISTRIBUTION / AVAILABILITY STATEMENT
Statement A

DISTRIBUTION STATEMENT A
Approved for Public Release
Distribution Unlimited

13. SUPPLEMENTARY NOTES
PA# ESC 04-0316 Published in IEEE Microwave and Wireless Components Letters, Vol. 15, No. 9, pp.567-569, Sept. 2005

14. ABSTRACT
We report free space, microwave measurements with varying incidence angle on a material that exhibits a negative index of refraction. The experiments measure the index of refraction directly, for normal incidence, on prisms with three different apex angles. Snell's Law and the measured index are then used to verify refraction for off-normal incidence. The metamaterials used to construct the prisms are comprised of parallel metallic layers of posts and either 'Greek key' spiral or split ring resonators. Despite this highly anisotropic composition, the measured refractive index was found to be nearly constant for exit angles up to 52 degrees off the normal to the prism hypotenuse. In addition to being the first free space experimental verification of Snell's Law off the optical axis for this type of metamaterial, the measurements prove that the common parallel plate structures of negative index metamaterials are not acting as guided wave media.

15. SUBJECT TERMS
negative index media, NIM, anisotropic media, left-handed materials, metamaterial, negative index of refraction

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** John Derov
a. REPORT Unclassified	**b. ABSTRACT** Unclassified	**c. THIS PAGE** Unclassified	UU	3	**19b. TELEPHONE NUMBER** *(include area code)* 781-377-2638

Free Space Measurements of Negative Refraction with Varying Angles of Incidence

J. S. Derov, *Member, IEEE*, B.W. Turchinetz, *Member, IEEE*, E. E. Crisman, A. J. Drehman, S. R. Best, *Member, IEEE*, and R. M. Wing

Abstract—**We report free space, microwave measurements with varying incidence angle on a material that exhibits a negative index of refraction. The experiments measure the index of refraction directly, for normal incidence, on prisms with three different apex angles. Snell's Law and the measured index are then used to verify refraction for off-normal incidence. The metamaterials used to construct the prisms are comprised of parallel metallic layers of posts and either 'Greek key' spiral or split ring resonators. Despite this highly anisotropic composition, the measured refractive index was found to be nearly constant for exit angles up to 52 degrees off the normal to the prism hypotenuse. In addition to being the first free space experimental verification of Snell's Law off the optical axis for this type of metamaterial, the measurements prove that the common parallel plate structures of negative index metamaterials are not acting as guided wave media.**

Index Terms—**Anisotropic media, left-handed materials, metamaterial, negative index of refraction, NIM.**

I. Introduction

THE theoretical study of negative refractive index or left-handed materials was introduced by Veselago [1] in 1968.

In his theoretical analysis, Veselago assumed a continuous and isotropic medium with negative values of both permittivity and permeability. The first measurement of negative index media (NIM) [2], based on earlier theoretical work [3], was made at microwave frequencies using metallic split ring resonators (SRRs) and posts etched onto opposite sides of printed circuit substrates that were subsequently interlocked in orthogonal planes. The medium was clearly neither isotropic nor continuous. More recently, experimentalists have used prisms with parallel, rather than orthogonal, planes of substrates to show negative index effects [4-7]. The orthogonal and parallel plane structures are sometimes referred to as "2D" and "1D" respectively.

All measurements of negative refraction in metamaterials reported thus far have used incident radiation propagating along the direction normal to the incident prism face. In addition, the electric field was polarized parallel and the magnetic field normal to the longitudinal substrate planes. Negative refraction occurs at the second, exit, surface where the incident radiation arrives at an angle to the prism/air interface. The measured angle of the output signal and the incident angle are used to calculate an index of refraction for the media as if it were isotropic. This convenience has become accepted by the scientific community. However, to date no experiments have reported the consequence of varying the angle of incidence on metamaterial prisms and tested the limits over which the isotropically derived index may be applied. Our work is the first to examine whether the NIM are effectively true refractive media in the sense of satisfying Snell's Law for all incidences.

This verification is important because the 1D structure is quite similar to metallic plate lenses and metallic delay lenses, which do not satisfy Snell's Law for off-axis incidence [8]. Such structures are commonly referred to by the term 'guided wave' [9] and could show an effect that might be construed as refraction. Guided wave effects however are not dependent on the angle of incidence to the structure and can, therefore, be separated from true refraction by varying that angle. Changing the apex angle of the prism without changing the incidence angle as in [10] is insufficient because propagation in the NIM is still along the optical axis. The change in path lengths through a prism with different apex could change the direction of constructive wave interference of the output signal, as in path length lenses, without true refraction.

The interaction of the anisotropic NIM with off-axis radiation is the subject of this investigation. We have used measurements at normal incidence angle on prisms of different apex angles and different unit cell designs to extract a value for the index of refraction of each configuration. That index value is used to predict the expected exit angle for varying incidence angles, assuming isotropic behavior. The exit angles were measured for oblique incidence angles and compared to those predicted from the index at normal incidence. Thus for the first time NIM structures have been shown by experiment to be true refractive media for off-axis propagation.

II. Experiment

These measurements use the 1D, parallel plane configuration for two different metamaterial designs. The first type is the familiar post and SRR. The prism was fabricated and supplied to us by Boeing Phantom Works. It is described in [4]. The second type, designed and built by our group, uses a rectangular spiral or Greek key in place of the SRR. The advantage of the Greek key is its lower resonant frequency for a given unit cell area. The unit cell of this type is shown in Fig. 1. The posts are joined top to bottom at adjacent cells to form continuous lines 15 cm long. Since this length is several wavelengths at the frequencies of interest, the lines produce negative permittivity without a ground contact [11].

The posts and keys are etched into 1-oz copper on 0.254 mm Rogers' 5880 substrate, with permittivity 2.2 at 10 GHz. Then the boards are cut into strips of varying widths and

Manuscript received August 23, 2004. This work was supported by the AFOSR and DARPA DSO.

The authors are with the U. S. Air Force Research Laboratory, Hanscom AFB, Massachussetts. Corresponding author: Beverly.Turchinetz@hanscom.af.mil.

stacked to form wedge shaped prisms. A prism top view is shown in Fig. 2. The orientation of the boards in the prism is vertical, i.e. out of the page of Fig. 2, and parallel to the normal of the entrance face. The posts on the boards are also vertical. The boards are spaced with 3.2 mm layers of Emerson and Cumings Eccosorb™ PP2 foam, which is essentially transparent to microwaves.

Prisms were constructed from the Greek key cell with 12, 18 and 27 degree apex angles. Because of the finite size of the unit cell, the exit face is stepped to produce the apex angles by increasing the board lengths by one unit cell every 5, 3 or 2 boards respectively. All prisms have 8 cells minimum thickness. The entrance face on the prisms is 15 cm by 15 cm. For comparison, we measured a positive index prism of solid Teflon with apex angle of 32 degrees. The Teflon prism was also constructed with steps on the hypotenuse; step size 3.3 mm every 5.3 mm.

The measurements were performed in free space using identical transmit and receive horns of dimension 20 cm by 20 cm. The horns have a 3dB beam width of 14 degrees. The transmit horn was mounted at a fixed position 3.73 m from the prism. The receive horn was mounted 2.58 m from the prism on an arm that permitted ±90 degrees rotation about the normal to the prism exit face. The prism itself was mounted on a platform that could be rotated to change its angle with respect to the transmitter. Absorbent baffles were placed around the prism to prevent direct communication between transmitter and receiver.

Figure 2 shows possible ray paths for the prism measurement. An incident ray normal (N) to the first face is not refracted and meets the second face at an angle equal to the apex angle. An ordinary (positive index) material would refract the ray to the opposite side of the exit normal. Negative index materials refract the ray to the same side of the normal. For convenience we label angles left of the normal as negative and right of the normal as positive, for both the entrance and exit rays. Note the signs of these angles are an artifact of the labeling convention and are not carried through the Snell's Law equations. The sign of the *index* is determined only by whether the refracted ray exits the interface on the same or on the opposite side of the normal as the incident ray.

III. Results

The prisms were first measured at normal incidence, with the electric field parallel to the posts and the magnetic field perpendicular to the plane of the substrates. The negative refraction frequency range for the Greek key type is 9.9 to 10.4 GHz. At various frequencies in the negative range for each prism, the index of refraction n was calculated using Snell's Law,

$$n\sin(\alpha)=\sin(\theta_r) \qquad (1)$$

where α, the prism apex angle, is also the angle of incidence at the second face and θ_r is the measured exit angle. The NIM indices ranged from −1.4 to −0.6 assuming the index for air is 1. The measured index value of 1.4 for Teflon agrees with the published value.

Three Greek key NIM prisms with different apex angles were constructed in order to investigate the consistency of the metamaterial construction technique and the metamaterial index. For an ordinary isotropic material, the index of refraction would of course be independent of the apex angle. Since the negative index is highly frequency dependent, the frequency at which the measured index equals -1 is a useful standard for comparison. For all three prisms, that frequency was between 10.1 and 10.2 GHz. The agreement within 1% in frequency shows consistency of the prism fabrication and of the on-axis index. It confirms the appropriateness of treating the metamaterial as an effective media for the special case of propagation along the optical axis. This confirmation was justification for proceeding to investigate off-axis incidence at the entrance face.

Measurements were made with oblique entrance angles at selected frequencies. The electric field remains parallel to the posts but the magnetic field is no longer perpendicular to the plane of the substrates. The results plotted in Fig. 3 show the exit angle, measured relative to the normal axis of the second (hypotenuse) face, as a function of entrance angle, measured from the normal to the first face.

The solid line with each data set is the exit angle plot calculated by using the index value measured at normal incidence at the specified frequencies. The calculation would be valid for isotropic material. Snell's Law is applied twice, once at each face, to solve for the exit angle. If n is the prism index at normal incidence, θ_i is the entrance angle, and α is the prism apex angle, then the equation for exit angle θ_r is

$$\theta_r=\sin^{-1}\left(\frac{|n|}{n}n\sin\left(\left(\sin^{-1}\left(\frac{1}{n}\sin\theta_i\right)+\frac{|n|}{n}\alpha\right)\right)\right). \qquad (2)$$

Comparison of the measured and calculated angles for the NIM in Fig. 3 shows good agreement for at least ±10 degrees about the normal incidence and modest deviation at higher angles. The split ring prism shows slightly more deviation than the Greek key at higher angles, possibly due to its double post design. Despite the anisotropy of the NIM, the measured index is nearly constant and quasi-isotropic across a broad range of incident angles. Measurements of the Teflon prism in comparison show, as expected, the opposite slope for positive refraction and negligible deviation from the calculation.

Some critics [12] have pointed out that near field measurements of negative refraction could be artifacts, explained by translation of the apparent beam center due to decreased loss through the thinner edge of the prism (see Fig. 2). For the receiver distance of 2.58 m in our setup, the 15 cm prisms subtend less than 4 degrees of arc. The observation of intensity maxima farther than 4 degrees from the transmit directions (at +12, +18, and +27 degrees from the exit normal for the different prisms) cannot be due to such translation. Also, some maxima were observed at exit angles in line with the thicker prism edge where there should be increased absorption loss. These observations support the identification of the signal as negative refraction.

Although the measured angles of the refracted signal closely followed the prediction, the measured magnitudes did not. It was anticipated that the magnitude would decrease away from normal incidence due to the anisotropy of the NIM. However, the rate of decrease was steeper than cosine squared. The width of the refracted signal and the height of the side lobes also increased away from normal incidence. Beyond the angles reported here for the NIM, the signal amplitude was

indistinguishable from the side lobes. Such increased attenuation has implications for off-axis incidence on 2D and 3-D material and is under further investigation.

IV. Conclusions

Despite the highly anisotropic and discontinuous nature of the resonator/post parallel plane structures, their behavior is quasi-isotropic with respect to negative refraction. The two NIM designs we evaluated obey Snell's Law for off-axis incidence and the measured refractive index is nearly constant over a range of incident angles. The experiments establish that guided wave phenomena are not the source of apparent refraction in such structures. Also, the measurements confirm the validity of the 'effective media' approach for calculations of NIM refraction. Our findings for 1D, parallel plane media broaden the applications for NIM that could be realized without the greater complexity of 2D orthogonal plane structures.

References

[1] V. G. Veselago, "The electrodynamics of substances with simultaneously negative values of ε and μ," *Soviet Physics Uspekhi,* vol. 10, pp. 509-514, 1968.

[2] R. A. Shelby, D. R. Smith and S. Schultz, "Experimental verification of negative index of refraction," *Science*, vol. 292, pp. 77-79, 2001.

[3] J. B. Pendry, A. J. Holden, D. J. Robbins and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microwave Theory Tech.*, vol. 47, pp. 2075-2084, 1999.

[4] R. B. Greegor, C. G. Parazzoli, K. Li, B. E. C. Koltenbah and M. Tanielian, "Experimental determination and numerical simulation of the properties of negative index of refraction materials," *Optics Express*, vol. 11, pp. 688-695, 2003.

[5] R. Ziolkowski, "Design, Fabrication and Testing of Double Negative Metamaterials," *IEEE Trans. Ant. Prop.*, vol. 51, pp. 1516-1529, 2003.

[6] E. Ozbay, K. Aydin, E. Cubukcu and M. Bayindir, "Transmission and Reflection Properties of Composite Double Negative Metamaterials in Free Space," *IEEE Trans. Ant. Prop.*, vol. 51, pp. 2592-2595, 2003.

[7] J. Huangfu, L. Ran, H. Chen, X. Zhang, K. Chen, T. Grzegorczyk, and J. A. Kong, "Experimental confirmation of negative refractive index of a metamaterial composed of Ω-like metallic patterns," *Appl. Phys. Lett.*, vol. 84, pp. 1537-1539, 2004.

[8] J. Brown, *Microwave Lenses.* New York: John Wiley and Sons, Inc., 1954, p. 62.

[9] R. E. Collin, Field Theory of Guided Waves. New York: McGraw-Hill, Inc., 1960, Chap. 12.

[10] A. A. Houck, J. B. Brock and I. L. Chuang, "Experimental Observations of a Left-Handed Material That Obeys Snell's Law," *Phys. Rev. Lett.*, vol. 90, pp. 137401-1-137401-4, 2003.

[11] W. E. Kock, "Metallic Delay Lenses," *Bell System Tech. J.*, vol. 27, pp. 58-82, 1948.

[12] M. Sanz, A. C. Papageorgopoulos, W. F. Egelhoff, Jr., M. Nieto-Vesperinas and N. Garcia, "Transmission measurements in wedge-shaped absorbing samples: An experiment for observing negative refraction," *Phys. Rev. E*, vol. 67, pp. 067601-1 - 067601-4, 2003.

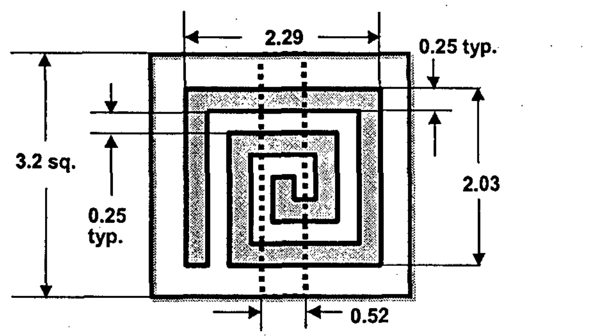


Fig. 1. Greek key unit cell with post underneath. Dimensions are mm.

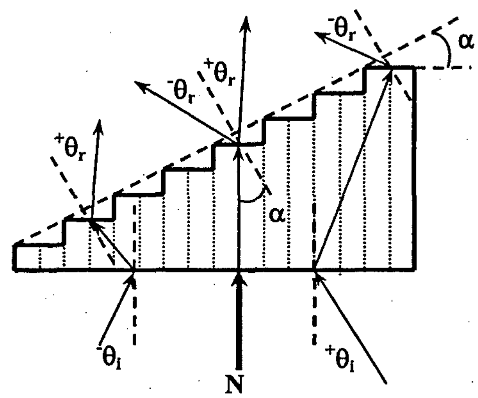
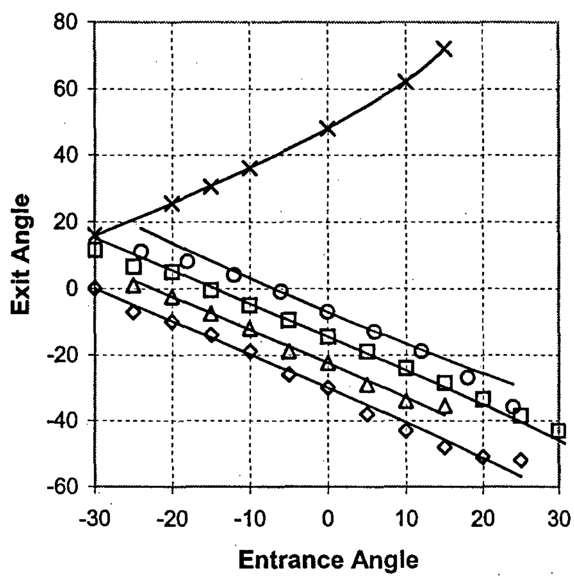


Fig.2. The prism measurement seen from above, showing the convention for labeling positive and negative angles in air.



	Prism	Apex	Index at 0°	Frequency (GHz)
x	Teflon	32°	+1.40	10.00
□	Greek key	12°	-1.20	10.05
Δ	Greek key	18°	-1.42	10.08
◊	Greek key	27°	-1.10	10.15
o	Split ring	12°	-0.59	14.50

— Lines are calculated using Index at 0° entrance angle

Fig. 3. Measured and calculated exit angle with varying entrance angles for the five prisms.